## edexcel

Mark Scheme (Results)
Summer 2014

Pearson Edexcel GCE in Mechanics 3 (6679_01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## PEARSON EDEXCEL GCE MATHEMATI CS

## General Instructions for Marking

1. The total number of marks for the paper is 75 .
2. The Edexcel Mathematics mark schemes use the following types of marks:
'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.
To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct e.g. in a moments equation, every term must be a 'force $x$ distance' term or 'mass $x$ distance', if we allow them to cancel ' $g$ ' s .
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned. e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity - this M mark is often dependent on the two previous M marks having been earned.

## ' A ' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

## 'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)
$A$ few of the $A$ and $B$ marks may be f.t. - follow through - marks.

## 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol $\sqrt{ }$ will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
- $\quad$ The answer is printed on the paper
- $\quad$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

6. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or $\sin$ ) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- dM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g=9.8$ should be given to 2 or 3 SF.
- Use of $\mathrm{g}=9.81$ should be penalised once per (complete) question.
N.B. Over-accuracy or under-accuracy of correct answers should only be penalised once per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads - if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.
N2L Newton's Second Law (Equation of Motion)
NEL Newton's Experimental Law (Newton's Law of Impact)
HL Hooke's Law
SHM Simple harmonic motion
PCLM Principle of conservation of linear momentum
RHS, LHS Right hand side, left hand side.

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 1. | $\begin{aligned} & R \sin \theta=m \times 4 r \mathrm{~s} \\ & R=\frac{3}{2} m g \\ & R \cos \theta=m g \\ & \frac{3}{2} m g \cos \theta=m g \\ & \cos \theta=\frac{2}{3} \end{aligned}$ $O C=4 r \cos \theta=4 r \times \frac{2}{3}=\frac{8}{3} r \text { oe }$ | M1A1A1 <br> M1A1 <br> M1(dep) <br> A1 <br> M1A1 <br> [9] |
| $\begin{array}{r} \text { M1 } \\ \text { A1 } \\ \text { A1 } \\ \text { ALT: } \\ \text { M1 } \\ \text { A1 } \\ \text { M1 dep } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 cso } \end{array}$ | Notes for Question 1 <br> for NL2 towards $C$ - Accept use of $v=\sqrt{\frac{3 g}{8 r}}$ and $a=\frac{v^{2}}{r}$ as a mis-read for LHS fully correct for RHS fully correct <br> Work in the direction of $R$ and obtain the same equation with $\sin \theta$ "cancelled". Give M1A1A1 if fully correct, M0 otherwise. <br> for resolving vertically <br> for the equation fully correct <br> for eliminating $R$ between the two equations Dependent on both above M marks for $\cos \theta=\frac{2}{3}$ <br> for attempting to use trig or Pythagoras to obtain $O C$ for $O C=\frac{8}{3} r$ |  |

## Alternative for Question 1

| M1A1A1 | $R \sin \theta=m \times a \times \frac{3 g}{8 r}$ |
| ---: | :--- |
| M1 A1 | $R \cos \theta=m g$ |
| M1 A1 | $\tan \theta=\frac{3 a}{8 r}$ |
| M1 | $\frac{a}{O C}=\frac{3 a}{8 r}$ |
| A1 | $O C=\frac{8 r}{3}$ |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| (a) | (At surface) $\frac{k}{R^{2}}=m g \Rightarrow k=m g R^{2}$ | M1A1 (2) |
| (b) | $m \ddot{x}=-\frac{m g R^{2}}{x^{2}}$ |  |
|  | $v \frac{\mathrm{~d} v}{\mathrm{~d} x}=-\frac{g R^{2}}{x^{2}}$ | M1 |
|  | $\int v \frac{\mathrm{~d} v}{\mathrm{~d} x} \mathrm{~d} x=-g R^{2} \int \frac{1}{x^{2}} \mathrm{~d} x \quad \text { or } \int \frac{\mathrm{d}\left(\frac{1}{2} v^{2}\right)}{\mathrm{d} x} \mathrm{~d} x$ |  |
|  | $\frac{1}{2} v^{2}=\frac{g R^{2}}{x}(+c)$ | DM1A1 |
|  | $x=\frac{5 R}{4}, v=\sqrt{\frac{g R}{2}} \Rightarrow c=-\frac{11 g R}{20}$ | DM1A1 |
|  | $v=00=\frac{g R^{2}}{x}-\frac{11 g R}{20}$ | DM1 |
|  | $x=\frac{20 R}{11}$ | A1 (7) |
|  |  | [9] |


|  | Notes for Question 2 |
| :---: | :---: |
| M1 | for $\frac{k}{R^{2}}=m g$. If not made clear that this applies at the surface of the Earth award M0 or $\frac{k}{x^{2}}=m g$ and $x=R$. |
| A1 cso | for $k=m g R^{2} *$ |
| (b) M1 | for using accel $=v \frac{\mathrm{~d} v}{\mathrm{~d} x}$ oe in NL2 with or w/o $m$ Minus sign not required. |
| M1 dep | for attempting to integrate both sides - minus not needed |
| A1 | for fully correct integration, with or w/o the constant. Must have included the minus sign from the start. |
| M1 dep | for using $x=\frac{5 R}{4}, v=\sqrt{\frac{g R}{2}}$ to obtain a value for the constant. Use of $x=\frac{R}{4}$ scores M0 Depends on both previous M marks |
| A1 | for $c=-\frac{11 g R}{20}$ |
| M1 dep | for setting $v=0$ and solving for $x$ Depends on 1st and 2nd M marks, but not 3rd |
| A1 cso | for $x=\frac{20 R}{11}$ |
| ALT: | By definite integration First 3 marks as above, then |
|  | Using limits $x=\frac{5 R}{4}, v=\sqrt{\frac{g R}{2}}$ |
| DM1 | Using limit $v=0$ |
| A1 | Correct substitution |
| A1 cso | for $x=\frac{20 R}{11}$ |
|  | NB: The penultimate A mark has changed position, but must be entered on e-pen in its original position. |

## Alternative for Question 2

Qu 2 (a):
Using $F=\frac{G M_{1} M_{2}}{x^{2}}$ with $x=R$ and one mass as mass of Earth:
$m g=\frac{G m M_{E}}{R^{2}}$
$G M_{E}=g R^{2} \Rightarrow F=\frac{m g R^{2}}{x^{2}} \Rightarrow F=\frac{k}{x^{2}}$ with $k=m g R^{2} *$
M1 Complete method A1 Correct answer

## Qu 2 (b):

By conservation of energy:
Work done against gravity $=\int_{\frac{5 r}{4}}^{z} \frac{m g R^{2}}{x^{2}} \mathrm{~d} x=\int_{\frac{5 r}{4}}^{z} m g R^{2} x^{-2} \mathrm{~d} x \quad$ M1
$=\frac{4 m g R}{5}-\frac{m g R^{2}}{z}$
DM1 (integration)A1 (correct)
Work-energy equation: $\frac{m g R}{4}=\frac{4 m g R}{5}-\frac{m g R^{2}}{Z}$
DM1A1
$z=\frac{20 R}{11}$


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| (a) | $\frac{3 m g x^{2}}{2 l}=2 m g x \sin \alpha$ | M1A1 <br> B1(A1 on e- <br> pen) |
|  | $\begin{aligned} & 3 x^{2}=4 x l \times \frac{3}{5} \\ & 5 x^{2}=4 x l \end{aligned}$ |  |
|  | $x=\frac{4}{5} l$ | DM1A1 (5) |
| (b) | $R=2 m g \cos \alpha\left(=\frac{8}{5} m g\right)$ | B1 |
|  | $\frac{3 m g}{2 l} \times \frac{4}{25} l^{2}=2 m g \times \frac{2}{5} l \times \frac{3}{5}-, \quad \mu \frac{8}{5} m g \times \frac{2}{5} l$ $6=12-16 \mu$ | M1A1ft, B1ft (A1 on epen) |
|  | $16 \mu=6 \quad \mu=\frac{3}{8}$ | DM1A1 (6) <br> [11] |


|  | Notes for Question 4 |
| :---: | :---: |
| (a) |  |
| M1 | for an energy equation with an EPE term of the form $\frac{k m g x^{2}}{l}$ and a GPE term. If a KE term is included it must become 0 later. |
| A1 | for a correct EPE term |
| B1 | for a correct GPE term. This can be in terms of the distance moved down the plane or the vertical distance fallen |
| M1 dep | for solving their equation to obtain the distance moved or using the vertical distance and obtaining the distance moved along the plane. |
| A1 | for $x=\frac{4}{5} l$ oe eg $x=\frac{12}{15} l$ |
| (b) |  |
| B1 | for resolving perpendicular to the plane to obtain $R=2 \mathrm{mg} \cos \alpha$. May only be seen in an equation. |
| M1 | for an work-energy equation with an EPE term of the form $\frac{\mathrm{kmgx}^{2}}{\mathrm{l}}$, a GPE term and the work done |
|  | against friction. The work term must include a distance along the plane. |
| A1 | for EPE and GPE terms correct and work subtracted from the GPE |
| B1 ft | for the work term ft their $R$ |
| M1 dep | for solving to obtain a value for $\mu$ |
| A1 cso | for $\mu=\frac{3}{8}$ oe inc 0.375 but not 0.38 |
|  | If $\boldsymbol{m}$ used instead of $\mathbf{2 m}$, assuming correct otherwise: |
| (a) | M1A1B0M1A0 (so 2 penalties for mis-read) |
| (b) |  |
| B1 | $R=m g \cos \alpha$ |
| M1, A1 | Equation, with EPE correct and $m g \times \frac{2}{5} l \times \frac{3}{5}$ |
| B1 ft | $\mu \frac{4 m g}{5} \times \frac{2}{5} l$ |
| DM1, A1 | $\mu=0$ |

## Alternative for Question 4

Qu 4: Using NL2:
(a)
$2 m a=2 m g \sin \alpha-\frac{3 m g x}{l}$
$2 v \frac{\mathrm{~d} v}{\mathrm{~d} x}=\frac{6 g}{5}-\frac{3 g x}{l}$
$v^{2}=\frac{6 g x}{5}-\frac{3 g x^{2}}{2 l},+c$
$v=03 g x\left(\frac{2}{5}-\frac{x}{2 l}\right)=0$
$x=\frac{4 l}{5}$
(b)
$R=2 m g \cos \alpha$
$2 v \frac{\mathrm{~d} v}{\mathrm{~d} x}=\frac{6 g}{5}-\frac{3 g x}{l}-\mu \frac{8 g}{5}$
$v^{2}=\frac{6 g x}{5}-\frac{3 g x^{2}}{2 l}-\mu \frac{8 g x}{5},+c$
$v=0 \quad x=\frac{2 l}{5} \quad \mu \frac{8}{5}=\frac{6}{5}-\frac{3}{2 l} \times \frac{2 l}{5}$ $\mu=\frac{3}{8}$

M1(equation and attempt integration)

A1, A1 (show $c=0$ )

M1 (set $v=0$ and solve)

A1

B1

M1 (eqn and int)A1, A1 (show $c=0$ )

M1 (set $v=0$ and solve)

A1

If SHM methods are used, SHM must be proved first.

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 5. |  |  |
| (a) | $\mathrm{Vol}=\pi \int_{0}^{\frac{\pi}{2}} y^{2} \mathrm{~d} x=\pi \int_{0}^{\frac{\pi}{2}} \cos ^{2} x \mathrm{~d} x$ | M1 |
|  | $=\pi \int_{0}^{\frac{\pi}{2}} \frac{1}{2}(\cos 2 x+1) \mathrm{d} x$ | M1 |
|  | $=\frac{\pi}{2}\left[\frac{1}{2} \sin 2 x+x\right]_{0}^{\frac{\pi}{2}}=\frac{\pi^{2}}{4}$ | DM1A1 (4) |
| (b) | $\pi \int_{0}^{\frac{\pi}{2}} y^{2} x \mathrm{~d} x=\pi \int_{0}^{\frac{\pi}{2}} x \cos ^{2} x \mathrm{~d} x$ | M1 |
|  | $=\pi \int_{0}^{\frac{\pi}{2}} \frac{1}{2} x(\cos 2 x+1) \mathrm{d} x$ |  |
|  | $=\frac{\pi}{2} \int_{0}^{\frac{\pi}{2}} x \cos 2 x \mathrm{~d} x+\frac{\pi}{2}\left[\frac{x^{2}}{2}\right]_{0}^{\frac{\pi}{2}}$ |  |
|  | $\frac{\pi}{2}\left[x \times \frac{1}{2} \sin 2 x\right]_{0}^{\frac{\pi}{2}}-\frac{\pi}{2} \int_{0}^{\frac{\pi}{2}} \frac{1}{2} \sin 2 x \mathrm{~d} x,+\frac{\pi^{3}}{16}$ | M1,B1 |
|  | $=0+\frac{\pi}{2}\left[\frac{1}{4} \cos 2 x\right]_{0}^{\frac{\pi}{2}}+\frac{\pi^{3}}{16}$ | DM1 |
|  | $=\frac{\pi}{8}[-1-1]+\frac{\pi^{3}}{16}=\frac{\pi^{3}}{16}-\frac{\pi}{4}$ | A1ft |
|  | $\bar{x}=\frac{\pi^{3}-4 \pi}{16} \div \frac{\pi^{2}}{4}=\frac{\pi^{2}-4}{4 \pi} \quad \text { or } \quad 0.467088 \ldots$ | M1A1 <br> (7) <br> [11] |


|  | Notes for Question 5 |
| :---: | :---: |
| (a) |  |
| M1 | for using $\mathrm{Vol}=\pi \int_{0}^{\frac{\pi}{2}} \cos ^{2} x \mathrm{~d} x$. If $\pi$ is missing here it must be included later to earn this mark. |
|  | Limits not needed |
| M1 | for using the double angle formula (correct) to prepare for integration. Formula must be correct. $\pi$ and limits not needed for this mark. |
| M1 dep | for attempting to integrate and substitute the correct limits (only sub of non-zero limit needed be to seen) dependent on both M marks. |
| A1 cso | for $\frac{\pi^{2}}{4} *$ (check integration is correct, answer can be obtained by luck due to the limits) |
| (b) | NB: The first 5 marks can be earned with or without $\pi$ |
| M1 | for using $\pi \int_{0}^{\frac{\pi}{2}} x \cos ^{2} x \mathrm{~d} x \quad \pi$ not needed; limits not needed. |
| M1 | for using the double angle formula (correct) and attempting the first stage of integration by parts |
| B1 | for $\frac{\pi^{3}}{16}$ or $\frac{\pi^{2}}{16}$ if $\pi$ not included. NB integration by parts not needed for this mark |
| M1 dep | for completing the integration by parts, limits not needed yet |
| A1 ft | $\text { for }=\frac{\pi}{8}[-1-1]+\frac{\pi^{3}}{16}=\frac{\pi^{3}}{16}-\frac{\pi}{4} \quad \text { or }=\frac{1}{8}[-1-1]+\frac{\pi^{2}}{16}=\frac{\pi^{2}}{16}-\frac{1}{4} \mathrm{ft} \text { on } \frac{\pi^{3}}{16}$ |
|  | for using $\bar{x}=\frac{\int \pi y^{2} x \mathrm{~d} x}{\int \pi y^{2} \mathrm{~d} x}$ The numerator integral need not be correct. |
| M1 | $\pi$ should be seen in both or neither integral |
|  | for $\bar{x}=\frac{\pi^{2}-4}{4 \pi} \quad$ oe eg $\frac{\pi}{4}-\frac{1}{\pi}$ or $0.467088 \ldots$ |
| A1 cso | Accept 0.47 or better but no fractions within fractions |
|  | (a) has a given answer, so the cso applies to the solution of (b) only. |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 6. |  |  |
| (a) | $\frac{1}{2} m U^{2}-\frac{1}{2} m v^{2}=2 m g a$ | M1A1 |
|  | $\begin{aligned} & T+m g=m \frac{v^{2}}{a} \\ & T=\frac{\left(m U^{2}-4 m g a\right)}{a}-m g \end{aligned}$ | $\begin{aligned} & \text { M1A1 } \\ & \text { DM1 } \end{aligned}$ |
|  | $T=\frac{m U^{2}-5 m g a}{a}$ | A1 |
|  | $T \geqslant 0 \Rightarrow U^{2} \geqslant 5 g a$ | DM1 |
|  | $U \geqslant \sqrt{5 a g}$ | A1 (8) |
| (b) | At top: $\quad T=\frac{9 m g a-5 m g a}{a}=4 m g$ | M1(either tension)A1 |
|  | At bottom: $\quad T^{\prime}-m g=\frac{m U^{2}}{a}$ | A1 |
|  | $k T=m g+\frac{9 m a g}{a}=10 \mathrm{mg}$ | DM1 |
|  | $k=\frac{10 m g}{4 m g}=\frac{5}{2}$ | A1 <br> (5) [13] |

## Notes for Question 6

(a)
for an energy equation, from the bottom to the top. A difference of KE terms and a PE term needed.
M1

A1
M1

M1 dep
A1
M1 dep

A1 cso

is set $=0$, award M0
for a correct expression for $T$
for using $T \geqslant 0$ to obtain an inequality for $U^{2}$ or $U$. Allow with $>$ Dependent on all previous M marks.
for $U \geqslant \sqrt{5 a g} *$ Watch square root! Give A0 if $>$ seen on previous line.
NB: The second and fourth M marks (and their As if earned) can be given together if $m g \leq m \frac{v^{2}}{a}$ is seen
(b)

M1
A1
A1
M1 dep
A1 cso
From bottom to a general point gets M0 until a value for $\theta$ at the top is used. $v^{2}=u^{2}+2 a s$ scores M0
for all terms correct (inc signs)
for NL2 along the radius at the top. Two forces and mass x acceleration needed.
Accel can be in either form here. But see NB at end of (a)
for a fully correct equation. Acceleration should be $\frac{v^{2}}{a}$ now.
for eliminating $v$ (vel at top) between the two equations. Dependent on both previous M marks. If $v$ is set $=0$, award M0
for obtaining an expression for the tension at the top or at the bottom, no need to substitute for $U$ yet.
Substitute for $U$ and obtain one correct tension ( 4 mg at top or 10 mg at bottom)
for the other tension correct
for using tension at bottom $=k \mathrm{x}$ tension at the top and solving for $k$
for $k=\frac{5}{2}$ oe


## Notes for Question 7

(a)

M1
A1
A1
(b)

## ALT for

## Alternative for Question 7

Qu 7 (d)
By reference circle:


Centre of circle is $O$
Angle $C O D=\theta \quad$ Angle $E O D=\alpha$
$\cos \theta=\frac{0.15 l}{0.3 l} \quad \theta=\frac{\pi}{3}$
M1
$\alpha=\pi-\frac{\pi}{3}=\frac{2 \pi}{3}$
B1
$\omega=\sqrt{\frac{2 g}{l}}$
time $=\frac{\alpha}{\omega}=\frac{2 \pi / 3}{\sqrt{\frac{2 g}{l}}}=\frac{2 \pi}{3} \sqrt{\frac{l}{2 g}}$
M1A1

PMT
r






#### Abstract

^[ \title{  <br>  <br>  } ]


